

Converse of Fuglede Theorem

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Abstract. The Fuglede-Putnam Theorem stands as a cornerstone in the spectral theory of bounded linear operators. One of its most notable corollaries states that the product of two commuting normal operators is also normal. The converse is not true in general, and thus, it is of interest to find certain conditions under which the normality of the product of two commuting operators implies the normality of each factor. Inspired by this problem, we investigate when subnormal operators T_1 and T_2 are quasinormal provided their product is quasinormal. Also, we obtain as a corollary that subnormal n -th roots of a quasinormal operator are quasinormal, and thus we answer the question asked by Curto et al. in [1]. Also, we give sufficient conditions for quasinormal (subnormal) operators T_1 and T_2 to be normal if their product is normal. Some results in multivariable theory of subnormal and quasinormal operators are also presented (see [2], [3]).

Keywords: subnormal operators; quasinormal operators; spherically quasinormal pairs; Fuglede Theorem.

References

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