

Matrices in Gauss-type quadratures for variable-sign weight functions

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Abstract. We consider a recently proposed Gauss-type quadrature formula with respect to a weight function that changes sign in the interior of the integration interval. An important step in its construction is to introduce a modifier function used to transform the given integral into a sum of one integral that does not cause a quadrature error and the other integral with a property that the points from the interior of the integration interval at which the weight function changes sign are the zeros of its integrand. Determining a modifier function requires solving an associated system of linear equations. For the same integral, different modifier functions can be chosen, and hence different linear systems can be obtained. From a theoretical perspective, only necessary is that the associated system has a solution, but from a computational perspective, it is also important that the associated system is not too ill-conditioned and that the structure of its matrix is as simple as possible. We analyze the conditions under which it is guaranteed to obtain, for instance, a system with a Vandermonde matrix or a system with an identity matrix. We also give examples where systems with an arbitrary matrix are obtained.

Keywords: Gauss quadrature formula; variable-sign weight function; linear system.

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