

Reverse and Forward Order Law for the (b,c) -inverse

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Abstract. In a ring R it is well known that if $a, w \in R^{-1}$, then aw is also invertible and $(aw)^{-1} = w^{-1}a^{-1}$. This property is called the reverse order law. As the reverse order law holds for the classical inverse, the topic of necessary and sufficient conditions for generalized inverses became very often in the past decades.

The similar property, $(aw)^{-1} = a^{-1}w^{-1}$, is known as the forward order law. Contrary to the reverse order law, even if a and w are both invertible, the forward order law is not valid in general.

Here we present results for the reverse order law for the (b, c) -inverse in a unital ring. An equivalent condition for this law to hold for the (b, c) -inverse is derived. Furthermore, the forward order law for the (b, c) -inverse in a ring with a unity is introduced for different choices of b and c . Moreover, as corollaries of obtained results, equivalent conditions for the reverse order law and the forward order law for the inverse along an element are derived.

Keywords: (b, c) -inverse, Inverse along an element, Reverse order law, Forward order law.

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