

(Non-)existence of Lagrangians in hyperkähler manifolds

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Abstract. The purpose of this review article is to convey two phenomena, the existence and non-existence of exact Lagrangian submanifolds inside hyperkähler manifolds that have holomorphic *contracting* \mathbb{C}^* -actions.

Lagrangians of the first kind, discovered in [1], arise as the minima of the moment map of the S^1 -parts of the \mathbb{C}^* -actions. They are exact Lagrangians, which, in the example of 4-dimensional A_n -resolutions generate the (compact) Fukaya Category. The same is expected in a much bigger generality called *hypertoric varieties*.

The non-existence of other exact Lagrangians, pioneered in [2] for *ADE resolutions* (that contain the aforementioned A_n -resolutions), comes from two results: (1) vanishing of a certain Floer-theoretic invariant called *symplectic cohomology* and (2) isomorphism between this invariant and its twisted counterpart, for the different symplectic form, obtained under the hyperkähler rotation of the former one. Result (1) is generalised in [3], covering in particular all known hyperkähler manifolds with \mathbb{C}^* -actions. Result (2) is still conjectural in this generality, but if correct, it would imply for instance that there are no Lagrangian spheres for all such spaces which are not in the (lowest) dimension 4.

Keywords: Lagrangian submanifolds; hyperkähler manifolds; symplectic cohomology.

References

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