

## Semi-Fredholm theory in unital $C^*$ -algebras

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**Abstract.** Axiomatic (semi-)Fredholm theory in unital  $C^*$ -algebras has been established by Kečkić and Lazović in [8] and by Ivković in [5]. In this talk, we shall present a new approach to this theory given in [6] and [7]. Next, we shall extend Fredholm theory in von Neumann algebras established by Breuer in [1] and [2] to spectral Fredholm theory in von Neumann algebras. We shall consider upper triangular operator matrices with coefficients in a von Neumann algebra and give the relationship between the generalized essential spectra (in the sense of Breuer) of such matrices and of their diagonal entries, thus generalizing in this setting the result by Djordjević in [3]. Finally, we shall introduce the concept of generalized Browder operators in von Neumann algebras and provide a characterization of these operators in terms of the property of being generalized Fredholm in the sense of Breuer and having 0 as an isolated point of the spectrum. Also, we shall illustrate this result by concrete examples. This is a continuation of the research in [4].

**Keywords:** Fredholm operator, Browder operator, von Neumann algebra, essential spectra

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