Existence of Large-Data Global Weak Solutions to Navier–Stokes–Fokker–Planck Systems

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Abstract. Since the pioneering contributions of Werner Kuhn, Hans Kramers and other scientists working at the interface of polymer chemistry and statistical physics during the first half of the twentieth century, kinetic models have been widely and successfully used to describe the motion of polymeric fluids. The aim of this talk is to review recent results concerning the mathematical analysis of these models. We focus in particular on questions of existence of large-data global-in-time weak solutions to kinetic models of dilute polymeric fluids — a system of nonlinear partial differential equations involving the compressible or incompressible Navier–Stokes equations, modelling the evolution of the velocity field and the pressure, coupled to the Fokker–Planck equation satisfied by the probability density function for the random configuration vectors associated with the directions of the backbones of noninteracting polymer molecules suspended in a Newtonian fluid. We shall highlight some nontrivial open problems related to the breakdown of weak compactness in L^1 and the appearance of the divergence of a symmetric positive semi-definite matrix-valued defect measure in the balance of linear momentum equation in the Navier–Stokes system. The key results discussed in the talk may be found in [1]–[5].

Keywords: Navier–Stokes–Fokker–Planck systems; global weak solutions; polymeric fluids.

References

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