Computer solution of an arithmetical problem

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Abstract. Consider the following problem from elementary algebra[1]: given the number n, determine all decimal numbers $\overline{a_n a_{n-1} \ldots a_1}$, $a_n > 0$, such that $\overline{a_n a_{n-1} \ldots a_1} = \sum_{i=1}^{n} a_i^n$. For example, if $n = 3$, then $371 = 3^3 + 7^3 + 1^3$. The author gives some solutions for $n \leq 10$, and asks if there are solutions if $n > 10$. Cominatorial explosion makes the solution of this problem difficult for $n > 10$. Another difficulty arises because the solution demands multyprecision arithmetic. V. Janković and M. Živković [2] show that there are solutions only if $n \leq 60$, and reduce the problem to find the numbers $n_j = |\{i : a_i = j\}|$, $0 \leq j \leq 9$, such that $\sum_{j=0}^{9} n_j = n$ and the number od decimal digits j of $\sum_{j=0}^{9} n_j j^n$ equals to n_j for all $0 \leq j \leq 9$. They list all the solutions for $n \leq 25$, obtained by backtracking algorithm, and show that the smallest n for which there are no solutions is $n = 12$. Here we give details and consider some improvements of the algorithm.

Keywords: multiprecision arithmetic, elementary algebra, backtracking.

References

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[2] **В. Јанковић, М. Живковић.** Примена рачунара приликом решавања неких математичких проблема, *Тангента*, 2024, 115, 27 - 35.