

Loop homology of polyhedral products and Golod rings

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Abstract. In the 1950s, J.-P. Serre proved that the Poincaré series of a commutative local Noetherian ring is component-wise bounded by a certain rational function depending on the Betti numbers of the Koszul complex and the minimum number of generators in the maximal ideal. In 1962, E.S. Golod showed that Serre's inequality turns into equality if and only if multiplication and all Massey products in Koszul homology of a local ring are trivial; such a local ring is called a Golod ring. J. Bakelin proved in 1982 that the Poincaré series of monomial rings are rational; among the monomial rings there is a well-known class of Stanley-Reisner rings (or face rings) of simplicial complexes.

In this talk, we will discuss how toric topology allows us to establish combinatorial, algebraic and topological conditions equivalent to Golodness and minimal non-Golodness of a face ring of a simplicial complex over any field. We are going to describe these two classes of Stanley-Reisner rings in terms of their Poincaré series, Koszul homology, and the structure of the Lie algebra on the loop homology of the corresponding moment-angle complexes. We will see how the theory of spaces with a compact torus action allows us to obtain topological interpretations of the algebraic properties of Poincaré series and Koszul homology of Stanley-Reisner rings, as well as to get some new results.

The talk is based on joint work with T.E. Panov (Moscow State University).

Keywords: Polyhedral product, Stanley-Reisner ring, Golod ring, Massey product, Poincaré series.

References

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