Translational regular variation and asymptotic equivalence

Dragan Djurčić

Faculty of Technical Sciences in Čačak, University of Kragujevac, 32000 Čačak, Serbia
e-mail: dragan.djurcic@gradina.ftn.kg.ac.rs

Danica Fatić

Faculty of Technical Sciences in Čačak, University of Kragujevac, 32000 Čačak, Serbia
e-mail: danicafatic@gmail.com

Valentina Timotić

Faculty of Philosophy in Pale, University of East Sarajevo, 71420 Pale, Bosnia and Herzegovina
e-mail: valentina.timotic@ffuis.edu.ba

Abstract. Consider the class of positive functions

$$\mathcal{F} = \{x = x(t), t > a, a > 0 : x(t) \to \infty \text{ as } t \to \infty\}.$$

We say that functions $x, y \in \mathcal{F}$ are *multiplicatively strongly asymptotic equivalent* if

$$\rho_1 = \lim_{n\to\infty} \frac{x(t)}{y(t)} = 1, \tag{1}$$

and *additively strongy asymptotic equivalent* if

$$\rho_2 = \lim_{t\to\infty} (x(t) - y(t)) = 0. \tag{2}$$

A measurable function $f : [a, \infty) \to (0, \infty)$, $a > 0$, is *translationally regularly varying* in the sense of Karamata (see, for instance, [1]) if for each $\lambda \in \mathbb{R}$

$$\lim_{t\to\infty} \frac{x(\lambda + t)}{x(t)} < \infty. \tag{3}$$

The class of such functions is denoted by $\mathsf{Tr}(\mathsf{RV}_\varphi)$.

In this paper we prove that

$$x(t)\rho_1 y(t),\ t \to \infty, \tag{4}$$

implies that

$$f(x(t))\rho_2 f(y(t)),\ t \to \infty, \tag{5}$$

is implied by $f \in \mathsf{Tr}(\mathsf{RV}_\varphi)$.

In connection with (4) and (5) see, for example [2].

Keywords: translational regular variation; additively strongy asymptotic equivalence; multiplicatively strongly asymptotic equivalence.

References

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[2] **D. Djurčić, A. Torgašev.** Strong Asymptotic Equivalence and Inversion of Functions in the Class Kc. *Journal of Mathematical Analysis and Applications*, 2001, 255, 383 - 390.