Global regularity of Weyl pseudo-differential operators with radial symbols in each phase-space variable

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Abstract. We analyse a class of pseudo-differential operators in the Gelfand-Shilov setting whose Weyl symbols are radial in each phase-space variable separately. Namely, the symbols are of the form

$$a_\vartheta(x,\xi) := a(2x_1^2 + 2\xi_1^2, \dots, 2x_d^2 + 2\xi_d^2),$$

where a is a measurable function on $\mathbb{R}^d_+ := \{r \in \mathbb{R}^d \,|\, r_j > 0,\, j = 1, \dots, d\}$ and has Gelfand-Shilov L^p-growths. We prove that the action of these pseudo-differential operators on a Gelfand-Shilov ultradistribution f can be given by a series of Hermite functions with coefficients that are explicitly computed in terms of the Laguerre coefficients of a and the Hermite coefficients of f. As a consequence, we give a characterisation of the functions a in terms of the growths of their Laguerre coefficients for which the Weyl quantisation of a_ϑ are globally Gelfand-Shilov regular.

Keywords: Pseudo-differential operators with radial symbols, Gelfand-Shilov regularity, Hermite expansions, Laguerre expansions.

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