

The Riesz Potential Type Operator in the Power-Weighted Variable Generalized Hölder Spaces Over a Metric Measure Space

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Abstract. Let $\mathcal{X} = (\mathcal{X}, d, \mu)$ be a metric measure space, where d and μ are the distance and measure on \mathcal{X} , respectively. To characterize \mathcal{X} , the following condition is imposed:

$$\forall x \in \mathcal{X} : \quad \mu[B(x, h)] \leq K h^N \quad \text{with} \quad h \rightarrow 0, \quad K > 0,$$

where $B(x, h)$ symbolizes an open ball, and $N > 0$ is real. The following integral operator is considered here as the Riesz potential type operator over an open bounded $\Omega \subset X$:

$$I^\alpha f(x) := \int_{\Omega} \frac{f(\sigma) d\mu(\sigma)}{d^{N-\alpha}(x, \sigma)}, \quad x \in \Omega, \quad 0 \leq \operatorname{Re} \alpha < 1.$$

This study is on the action of I^α between the variable generalized Hölder spaces $H^{\omega(\cdot)}$, which were introduced in [1]. These spaces are defined through the following condition:

$$\omega_\Omega(f, x, t) \leq c \omega(x, t), \quad 0 < c < \infty,$$

formulated in terms of the local modulus of continuity, that is

$$\omega_\Omega(f, x, t) := \sup_{y \in \Omega \cap B[x, h]} |f(x) - f(y)|, \quad x \in \Omega, \quad h > 0,$$

or the minimum majorant of ω_Ω with the required properties. Specifically, we consider the conditions under which I^α is bounded from

$$H^{\omega(\cdot)}(\Omega, w) := \{f : \quad w f \in H^\omega(\Omega), \quad (w f)(a) = 0\}$$

with the power weight

$$w(x) = d^\nu(x, a), \quad a, x \in \Omega, \quad 0 < \operatorname{Re} \nu \leq 1,$$

to $H^{\omega_\alpha(\cdot)}$ characterized by $\omega_\alpha(x, h) = h^{\operatorname{Re} \alpha} \omega(x, h)$.

This study continues the approach of [2], where the Zygmund type estimates were obtained, and, based on these, theorems on the action of $I^{\alpha(\cdot)}$ over a quasi-metric space were proved in the weightless and special weight cases. The results are presented in terms of the Bary–Stechkin classes, as well as the Orlicz indices.

Keywords: fractional calculus, Riesz potential, variable generalized Hölder space

References

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