

Dynamical simulations and bisimulations between weighted finite automata over the field of real numbers

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Abstract. Weighted automata belong to the basic computation models in computer science. They can be understood as an extension of conventional automata in which transitions and states carry numerical or other values of different types, called weights. We will investigate simulations and bisimulations for weighted finite automata that take weights in the field of real numbers. Basic properties of simulations and bisimulations for weighted automata over real numbers will be presented. The proposed concept of bisimulations is compared with other concepts of bisimulations for weighted finite automata over the field of real numbers that can be encountered in the literature (cf. [1–3]).

The problem of testing the existence of simulations and bisimulations for automata over real numbers, their modeling in time-varying case and their computation is discussed. Theoretical background of the investigation is given in [4], in the general context of weighted finite automata over a semiring. Forward-backward bisimulation (fbb) and backward-forward bisimulation (bfb) between two weighted finite automata (WFA) require a certain system of homogeneous Sylvester equations and two vector equations. In this way, the problem is transferred to linear algebra models over the real numbers. These systems of vector and matrix equations have no solution in the general case. Our intention is to apply continuous-time dynamical systems known as the Zeroing Neural Network (ZNN) approach to solve approximately such vector-matrix systems. Zhang Neural Network or Zeroing Neural Network (ZNN) dynamics has been proposed in 2001 for solving time-varying problems. The ZNN's architecture is based on setting each element of to 0. This is accomplished using the continuous-time learning regulation that arises from the establishment of error matrix or vector equations, known as error functions. Applying block matrix representation of involved error functions in common with the vectorization and the Kronecker product, it is possible to transform the model based on systems of vector and matrix equations into a single system of linear equations with mass matrix given in block form. Then the solution of generated system of linear equations is obtained using the matrix pseudoinverse.

On the other hand, forward and backward simulation between WFA is based on certain systems of matrix and vector inequalities. Typically, such systems of inequalities possess infinite number of solutions. Usage of ZNN dynamics is usable for choosing the optimal solution. In this way, approximate solutions are considered in time-varying form. Proposed models are able to determine approximate solutions or to choose best solution between an infinite number of possible solutions..

The structure of developed ZNN models is based on composite models with various matrix or vectors error functions. Based on its global convergence without conditions, the proposed ZNN dynamical system is aimed to force the convergence of involved error functions to zero. Strategy of development of ZNN dynamics based on several error functions has been exploited in several research papers, involving [5–9].

Convergence analysis of proposed dynamical models is considered. Numerical examples are performed with different initial state matrices.

Keywords: Weighted finite automata, zeroing neural network, simulation, bisimulation, Sylvester matrix

equations.

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