# Bisimulations for weighted finite automata over the field of real numbers

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**Abstract.** Simulation and bisimulation relations are powerful tools used in many areas of computer science to match moves and compare the behaviour of various computing systems, such as labelled transition systems and automata, as well as to reduce the number of states of these systems. By moving from traditional Boolean-valued systems to quantitative ones, a need arise for both simulations and bisimulations to be quantitative, to be modeled with matrices whose entries should provide a measure of the connectivity of states of the considered systems.

Our approach consists of defining quantitative simulations and bisimulations as matrices that are solutions of certain systems of matrix inequalities and equations. Such an approach was applied for the first time in [4], where quantitative simulations and bisimulations between fuzzy finite automata were introduced and their basic properties were examined, while in [5] algorithms were developed for testing the existence of simulations and bisimulations of a given type. The same algorithms compute the greatest simulations and bisimulations, in cases where they exist. Then the same approach was applied to the study of simulations and bisimulations for non-deterministic automata [3], weighted finite automata over an additively idempotent semiring [8], and max-plus automata [7], as well as for weighted finite automata over an arbitrary semiring [6], which encompass all the previous ones. It turned out that almost identical methodology can also be applied to social networks [9] (in positional analysis and blockmodeling) and Kripke models of fuzzy multimodal logics [10, 11].

Here we use the same approach in defining simulations and bisimulations for weighted finite automata over the field of real numbers. We will present the basic properties of simulations and bisimulations for this type of weighted automata and show that there are important differences in comparison with the previously mentioned types of automata. We will also compare our concept of bisimulations with other concepts of bisimulations for weighted finite automata over the field of real numbers that can be encountered in the literature (cf. [1, 2, 12]). The problem of testing the existence of simulations and bisimulations for this type of automata and their computation will be discussed in a separate lecture.

**Keywords:** Weighted finite automaton, containment problem, equivalence problem, simulation, bisimulation, matrix inequations and equations.

**Acknowledgement.** The authors acknowledge the support of the Science Fund of the Republic of Serbia, Grant No. 7750185, Quantitative Automata Models: Fundamental Problems and Applications – QUAM.

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