Idempotent-aided factorizations of matrices over a field

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Abstract. We define an idempotent-aided factorization of a matrix D, with the help of an idempotent matrix E of the same rank as D, as the representation of D in the form $D = UV$, where the matrix U has the same rank as D and the same null space as E, while the matrix V has the same null space as D and the same range as E. Such factorizations can be viewed as a natural generalization of full rank factorizations.

Here we provide three efficient algorithms for determining idempotent-aided factorizations of matrices over a field, as well as the fourth one that determines the so-called canonical idempotent-aided factorization. We also apply those algorithms in the construction of algorithms for testing the existence and computing group inverses and (B, C)-inverses of matrices over a field.

We also show that the concept of idempotent-aided factorization can be defined in an even more general context – in an arbitrary semigroup. We prove that every regular element of a semigroup has an idempotent-aided factorization with respect to an arbitrary idempotent from the Green's $\mathscr{D}$-class of that element.

Keywords: Matrix factorization; Idempotent-aided factorization; Full rank factorization; Group inverse; (B, C)-inverse.

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