

## Capillary motion through a pipe with a variable cross section

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**Abstract.** Washburn's equation is one of the widely used models for describing rise of a liquid column in vertical narrow pipes. In this work, we extend the existing model by introducing the variable radius. Similar problem was recently studied in [2], but inertial and gravitational effects were neglected.

Governing equation is derived from the momentum balance equation in integral form, under the assumption of Poiseuille flow and no-slip boundary condition at the pipe wall. We show that asymptotic approach to equilibrium may be monotonic or oscillatory with respect to the critical parameter. This also holds true in the case of the constant radius. [1] We impose conditions under which certain effects (i.e. gravity, inertia or viscosity) may be neglected in the scaled equation.

**Keywords:** capillary motion, viscous fluids, asymptotic analysis

### References

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