

## **$L^p$ norm of truncated Riesz transform and an improved dimension-free $L^p$ estimate for maximal Riesz transform**

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**Abstract.** We prove that the  $L^p(\mathbb{R}^d)$  norm of the maximal truncated Riesz transform in terms of the  $L^p(\mathbb{R}^d)$  norm of Riesz transform is dimension-free for any  $2 \leq p < \infty$ , using integration by parts formula for radial Fourier multipliers. Moreover, we show that

$$\|R_j^* f\|_{L^p} \leq \left(2 + \frac{1}{\sqrt{2}}\right)^{\frac{2}{p}} \|R_j f\|_{L^p}, \quad \text{for } p \geq 2, \quad d \geq 2.$$

As by products of our calculations, we infer the  $L^p$  norm contractivity of the truncated Riesz transforms  $R_j^t$  in terms of  $R_j$ , and their accurate  $L^p$  norms. More precisely, we prove:

$$\|R_j^t f\|_{L^p} \leq \|R_j f\|_{L^p}$$

and

$$\|R_j^t\|_{L^p} = \|R_j\|_{L^p},$$

for all  $1 < p < +\infty$ ,  $j \in \{1, \dots, d\}$  and  $t > 0$ . Along with proved results, we will discuss possible extensions and generalizations.

### **References**

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