LIPSCHITZ CONTINUITY FOR FUNCTIONS SATISFYING α LAPLACIAN-GRADIENT INEQUALITY

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Abstract. We say that function f satisfies the α Laplacian-gradient inequality on the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ if there exist positive constants a, b such that

$$(1 - |z|^2)^\alpha |\overline{L_\alpha} f(z)| \leqslant a|\nabla f(z)|^2 + b \text{ for every } z \in \mathbb{D}.$$

Here $\overline{L_\alpha} = D_{\overline{z}}((1 - |z|^2)^{-\alpha} D_z)$. Let G be domains in $\mathbb{C}$ with C^2 boundary. Based by One of results obtained in [1] we will prove that, if a quasiconformal diffeomorphism $f : \mathbb{D} \to G$ satisfies the α Laplace-gradient inequality, then f is Lipschitz. The proof of this result is based on the Flattening the boundary method, with some use of continuity properties of Riesz potentials. Method used for proving the main result of this article is sometimes referred as Bootstrap argument.

Keywords: Poisson kernel; Green function; Generalized harmonic functions.

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