Metric dimension of hypercube and coin weighing problem

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Abstract. In this paper we consider the connection between metric dimension problem and minimal doubly resolving set problem of hypercube graph with coin weighing problem. Metric dimension of hypercube β_n is the minimum cardinality of a resolving set [3, 8], and ψ_n is the minimum cardinality of a doubly resolving set of hypercube graph Q_n [1]. Coin weighing problem posed by Söderberg and Shapiro can be defined as follows: for n coins, each with one of two distinct weights, determine the weight of each coin with the minimum number of weighings (f_n) [9]. It is known that f_n differs from β_n by at most one [5, 7]. We have shown that it holds more precisely: $f_n \leqslant \beta_n \leqslant f_{n-1} + 1$. Also, we have shown that equality $\psi_n = f_n + 1$ holds and, as a consequence, $\beta_{m+n} \leqslant \beta_m + \beta_n$. The last inequality shows that well-known hypothesis $\beta(G\Box H) \leqslant \beta(G) + \beta(H)$ is true in the case of hypercube graph.

Keywords: graph theory, metric dimension; doubly resolving set; coin weighing; hypercube graph.

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