On estimating convergence for Pickard sequences in quasi b-metric and rectangular quasi b-metric spaces

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Abstract. The iterative sequence $\{x_n\}$ with arbitrary x_0 converges (under the assumptions in Banach's Fixed Point Theorem [1]) to the unique fixed point x^* of contraction T. Error estimates are the prior estimate

$$d(x_n, x^*) \leqslant \frac{\lambda^n}{1-\lambda} d(x_0, x_1),$$

and the posterior estimate

$$d(x_n, x^*) \leqslant \frac{\lambda}{1-\lambda} d(x_{n-1}, x_n).$$

Since the introduction of quasi b-metric spaces by M. H. Shah, N. Hussain [4], many standard results have been generalized and elaborated in some papers. Recently, Z. Mitrović, I. Aranđelović, V. Mišić, H. Aydi and B. Samet [3] (see also [2]), proved that the sequence $\{x_n\}$ in quasi b-metric space with the contractive condition

$$d(x_{n+1}, x_n) \leqslant \lambda d(x_n, x_{n-1}),$$

for all $n \in \mathbb{N}$, where $\lambda \in [0, 1)$ is Cauchy sequence.

In this paper, using the same contractive condition as in [3], we give an estimate of the $d(x_n, x^*)$ for a sequence $\{x_n\}$ in a quasi b-metric space. In addition, we give another proof for the convergence of a sequence $\{x_n\}$. Examples of estimation for Banach's, Kannan's, and Reich's fixed point theorems are given. Following the same idea, in the second part of talk (paper) we shall introduce notion of the rectangular quasi b-metric spaces and present estimation in this class of spaces. At the end, we give some open problems where research can be continued.

Keywords: Cauchy sequence; quasi b-metric spaces; rectangular quasi b-metric spaces.

References

[1] **S. Banach**, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, *Fund. Math.*, 1922, 3, 133-181.

[2] **R. George , I. Aranđelović , V. Mišić, Z. Mitrović**, Some Fixed Points Results in b-Metric and Quasi b-Metric Spaces, *Journal of Function Spaces* Volume 2022, Article ID 1803348, 6 pages.

[3] **Z. Mitrović, I. Aranđelović, V. Mišić, H. Aydi, B. Samet**, On d^* - complete topological spaces and related fixed point results, *Mathematics* 8, 2020, article 1447.

[4] **M. H. Shah, N. Hussain**, Nonlinear contractions in partially ordered quasi b-metric spaces, *Commun. Korean Math. Soc.*, 2012, 27:1, 117–128.