Numerical approximation of one dimensional fractional transmission problem

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Abstract. Fractional partial differential equations (FPDEs) have attracted significant attention in recent years, owing to their diverse applications across numerous scientific and engineering domains. Often, fractional-order models prove to be more suitable than their integer-order counterparts, as fractional derivatives and integrals facilitate the description of memory properties inherent in various materials and processes. In this context, investigation has been undertaken on a fractional-in-time transmission problem spanning two disjoint intervals. An a priori estimate has been established for its weak solution within a suitable Sobolev-like function space. The study delves into the well-posedness of an interface problem associated with this equation, demonstrating its stability within corresponding Sobolev-like function spaces. Furthermore, a finite difference scheme has been developed to approximate this problem, accompanied by a thorough analysis of its properties. An estimation of the convergence rate has been derived, aligning with the smoothness characteristics of the input data. A proposed difference scheme has been put forth and validated through several numerical examples.

Keywords: fractional derivative; transmission; sub-diffusion; finite differences; convergence rate.

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