Supergeometry and supersymmetries: an introduction

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Abstract. A "geometrical space" is a topological space endowed with a sheaf of commutative algebras: choosing different local models for the space yields different kinds of geometry: (real) differential geometry, (complex) analytic/holomorphic geometry, and algebraic geometry of schemes. In the last case, the sheaf-theoretic perspective can also be replaced by functorial point of view, where each space is replaced by its functor of points. In these geometries, symmetries are encoded into spaces which are also groups: this yields (real or complex) Lie groups and group-schemes; when restricting to infinitesimal symmetries, one considers Lie algebras instead.

Supergeometry is the outcome of playing the above game with commutative algebras replaced by commutative *super*algebras — i.e., $\mathbb{Z}_2$–graded algebras whose homogeneous elements commute or anticommute with each other. Their symmetries are formalized by the notions of (real or complex) *Lie supergroup* and of *supergroup-scheme*, and infinitesimal symmetries by that of *Lie superalgebra*.

In this talk I will introduce the basics of supergeometry and its symmetries, mainly stressing their algebraic version, i.e. the *algebraic* supergeometry of superschemes and supergroup-schemes, following the functor of points perspective. Besides introducing the basic definitions, I will focus onto the link between supergroups and Lie superalgebras, and finally I will present the technique of studying supergroups via super Harish-Chandra pairs.

Keywords: Superalgebra, Supergeometry, Algebraic Supergroups, Lie Superalgebras.

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